The Outline of A New Solution to The Liar Paradox

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Abstract: First I show some examples of the liar paradox. Then I introduce roughly the Tarski's hierarchy theory and point out the main disadvantage of this approach. And then I give a new proposal which can avoid the disadvantage met in Tarski's proposal. And I discuss some properties of my proposal such as the soundness, the completeness and the relation of my proposal and that of Tarski's. At last I show how to solve the semantic paradoxes by my proposal.

Key Words: Liar paradox, truth, hierarchy, restricted quantifier, (T) schema

The liar paradox is an old problem. There are many solutions to this problem. The most famous approach is Tarski's hierarchy theory. However, there are some objections to his proposal. In this paper I will give a solution to the liar paradox which can avoid the main objection to Tarski's proposal.

1 The Problems

The liar paradox is a set of problems. In this section I will show some examples of liar's paradox which represent the most famous problems we will meet when discussing this issue, and also I will use them to check my solution to the liar paradox in the third section.

Example (1): Example (1) is not true.

What is the value of this sentence? In the common way, according to (1) itself, if it is true it will be false, if it isn't true it will be true. Strictly, using the modern logic we have the deduction as follows[1]

1. (1) = (1) is not true	Given
2. (1) is not true	Hypothesis
3. *(1) is not true* is not true	1 and 2, by Intersubstitutivity
4. Not that (1) is not true	3, by (T)[2] and intersubstitutivity
5. (1) is not true and not that (1) is not true	2 and 4, by conjunction
6. (1) is true	2–5, by reductio ad absurdum
7. *(1) is not true* is true	1 and 6, by Intersubstitutivity
8.(1) is not true	7, by (T)
9. (1) is true and not that (1) is true	6 and 8, by conjunction

Example (2): A: B is not true.
B: A is true.

[1] This strict proof comes from Professor Wenfang Wang's manuscript.

Using the similar inference we can get that: If A is true then B is not true and hence A is not true; if A is not true then it is not true that B is not true, so B is true, hence A is true. So A is true iff A is not true, i.e. A is true and A is not true. The same is true of the example (5) when we suppose it is a paradox.

Example (3), Curry paradox: let K be equivalent to the next sentence

True(<K>)→ The earth is flat.

Here <K> is the name of the sentence K. Letting "A↔B" abbreviate "(A→B)∧(B→A)" and "⊥" abbreviate "the earth is flat", we now argue as follows:

1. K→(True(<K>)→ ⊥)	By construction of K
2. True(<K>)↔(True(<K>)→ ⊥)	1, by (T) and Intersubstitutivity
3. True(<K>)→(True(<K>)→ ⊥)	Left to Right of 2
4. (True(<K>)∧True(<K>))→ ⊥	3, by Importation
5. True(<K>)→ ⊥	4, by Intersubstitutivity ($\phi \wedge \phi \leftrightarrow \phi$)
6. (True(<K>)→ ⊥) → True(<K>)	Right to Left of 2
7. True(<K>)	5 and 6, by modus ponens
8. ⊥	5 and 7, by modus ponens

It looks as if all the liar paradoxes contain self-reference from the examples listed above. However, it's not the case, there is another example which shows that we can get a paradox without using self-reference.

Example (4) Yablo's paradox[3] imagine an infinite sequence of sentences (S1), (S2), (S3).... Each sentence claims that every subsequent sentence is untrue:

(S_1) for all k>1, S_k is untrue

(S_2) for all k>2, S_k is untrue

(S_3) for all k>3, S_k is untrue

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Suppose for contradiction that some S_n is true. Given what S_n says, for all k>n, S_k is untrue. Therefore (a) S_{n+1} is untrue and (b) for all m>k+1, S_m is untrue. By (b), what S_{n+1} says is in fact the case, whence contrary to (a) S_{n+1} is untrue. So every sentence S_n in the sequence is untrue. But then the sentences sequent to any given S_n are untrue whence S_n is true after all. Hence for any S_n, S_n is true iff it is not true. Obviously there isn't self-reference in this example, however, a liar-like paradox still appears.

Example (5): contingent liar. Consider the following statement made by Jones:

(1) Most of Nixon's assertions about Watergate are false.

Suppose, however, Nixon's assertions about Watergate are evenly balanced between the true and the false, except for one problematic case,

[2] Here (T) is Tarski's (T) Schema: x is true iff ϕ, where x is the name of ϕ.

[3] Cf. Yablo, Stephen, "Paradox without Self-reference", *Analysis*, Vol. 53: pp 251-252.

(2) Everything Jones says about Watergate is true.

Suppose that (1) is Jones's sole assertion about Watergate. Then (1) and (2) are both paradoxical: they are true if and only if they are false. However, if Jones says other sentences about Watergate and there are false sentences among them, then (1) is true, and (2) is false, and there is no paradox. So the contingent paradox means whether a sentence is a paradox or not depends on environment.

Example (6) Tarski's undefinability theorem. In fact, this example is not a liar paradox. But it is a very important issue about the liar paradox. Nearly every solution to liar's paradox has to be consistent with this theorem. For a logic L, which is rich enough to contain arithmetic, if B(v) is any formula in the language of arithmetic with v as the only free variable, then there is a sentence Q in that language such that $\vdash \rightharpoonup (\phi \leftrightarrow B(<\phi>))$[4]

Next, in section 2 I will first give an introduction to the Tarski's proposal, and then show the main disadvantage of this proposal. Then in section 3 I will present my proposal which can solve the problems mentioned in the above and avoid the main disadvantage of Tarski's proposal.

2 Tarski's proposal

Tarski claims that the natural language is ambiguous so we need artificial language. Then he distinguishes object language and metalanguage. Because of the famous Tarski's undefinability theorem the truth of the object language can be said only in the metalanguage. Metalanguage is relative, so there is a sequence of metalanguages. Besides, he claims that any truth definition must satisfy every substitution of the form: x is true iff ϕ. Here x is the name of the sentence ϕ. This form is called (T) schema or T biconditional.

Formally, $\mathscr{L}_0$ contains the following primitive predicate symbols: P^1_1, P^1_2, P^1_3...P^n_1, P^n_2, P^n_3... and Q_1, Q_2, Q_3...[5]; the function symbols: f_1, f_2, f_3... and the constants: t_1, t_2, t_3.... The formulas of $\mathscr{L}_0$ are built up by the usual operations of the first-order predicate calculus. This language cannot contain its own truth predicate, so a metalanguage $\mathscr{L}_1$ containing a truth predicate T_0 is needed to talk about the sentences of $\mathscr{L}_0$ which are true. $\mathscr{L}_1$=$\mathscr{L}_0 \cup \{T_0\}$. The formulas of $\mathscr{L}_1$ are defined as usual. The process can be iterated, leading to a sequence {$\mathscr{L}_0$, $\mathscr{L}_1$, $\mathscr{L}_2$, $\mathscr{L}_3$,...} of languages, each with a higher predicate T_n for the preceding language $\mathscr{L}_{n-1}$. $\mathscr{L}_{Tarski}=\cup\{\mathscr{L}_n : n< \omega\}$. The semantics of $\mathscr{L}_{Tarski}$ is as that of predicate logic except that $\sigma(T_n(t))=1$ iff $\sigma(t)=\ulcorner\phi\urcorner$ and $\sigma \models \phi$, where $\ulcorner\phi\urcorner$ is the Gödel number of the sentence ϕ of the language $\mathscr{L}_m$, m<n. Instead of the constant t, from now on I use $\overline{\ulcorner\phi\urcorner}$ to refer to the Gödel number of the sentence ϕ. Then the (T) schema splits ino $T_n(\overline{\ulcorner\phi\urcorner}) \leftrightarrow \phi$, n<$\omega$, and it is valid in the arithmetic models.

The common and most famous objection to Tarski's approach is that it fragment the concept of truth. And hence the (T) schema divided into many (T_n) schemas. But the opponents claim that there is only one truth predicate rather than many $true_n$s.

[4]For more details cf. Hartry Field, *Saving Truth from Paradox*, Oxford University Press, 2008, Chapter 1.

[5]Adding these predicate symbols is not necessary, but in order to talk about the relation between Tarski's proposal and mine I add them

3 My proposal

In this section I will give my approach to the liar paradox. This proposal can avoid the main objection to Tarski's proposal. In my proposal there is only one truth predicate T rather than many T_ns. And the semantics in my proposal is bivalent rather than many-valued as shown in Kripke's proposal or other many-valued logic proposals.

3.1 Syntax

Given a limit ordinal $\omega \leqslant \gamma < 2^{\omega}$. Let $\mathscr{L}_0$ be the language including predicate symbols $P^1_1, P^1_2, P^1_3 ... P^n_1, P^n_2, P^n_3$...and constants $c^0_{\perp 0}, c^0_{\perp 1}, c^0_{\perp 2}$... All the n-place function symbols are treated as n+1-place predicate symbols in the usual way, so there are no function symbols. The language $\mathscr{L}_0$ is in some sense[6] adequate to arithmetic. Besides in the syntax there is no common quantifiers $\forall$ and $\exists$, instead, there are many $\forall^{\alpha}$s and correspondingly there are many $\exists^{\alpha}$s, $\alpha < \gamma$.

$\mathscr{L}_1 = \mathscr{L}_0 \cup \{c^1_{\phi 0} : \phi$ is a formula of $\mathscr{L}_0$ and 0 means ϕ is an open formula $\} \cup \{c^1_{\phi 1} : \phi$ is a formula of $\mathscr{L}_0$ and 1 means ϕ is a closed formula $\} \cup \{$ T $\}$

$\mathscr{L}_{n+1} = \mathscr{L}_n \cup \{c^{n+1}_{\phi 0} : \phi$ is a formula of $\mathscr{L}_n$ that doesn't appear before, 0 means ϕ is an open formula $\} \cup \{c^{n+1}_{\phi 1} : \phi$ is a formula of $\mathscr{L}_n$ that does not appear before, 1 means ϕ is a closed formula $\}$

For limit ordinal $\alpha < \gamma$, $\mathscr{L}_{\alpha} = \cup\{\mathscr{L}_{\beta} : \beta < \alpha\}$

$\mathscr{L}_T = \cup\{\mathscr{L}_{\beta} : \beta < \gamma\}$

Next we define formulas and sentences of $\mathscr{L}_T$ recursively. Since we don't have function symbols, the terms in our language contain only constants and variables.

Formu_0: $P^i_j c^0_{\perp k_1} \ldots c^0_{\perp k_i}$ is a formula
$P^i_j x_{k_1} \ldots x_{k_i}$ is a formula
If ϕ is a formula then $\neg \phi$ is a formula
If ϕ and φ are both formulas then $\phi \vee \varphi$ is a formula
If ϕ is a formula then $\forall^{\alpha}\phi$ is a formula, $\alpha < \gamma$
$\wedge, \rightarrow, \leftrightarrow$, and $\exists^{\alpha}$ can be defined as usual.

Formu_{n+1}: If ϕ is a formula in Formu_n then it is a formula in Formu_{n+1}
$Tc^i_{\phi j}$ is a formula where $c^i_{\phi j}$ is a constant in $\mathscr{L}_{n+1}$
$P^i_j c^{h_1}_{\phi k_1} \ldots c^{h_i}_{\chi k_i}$ is a formula where $c^{h_1}_{\phi k_1} \ldots c^{h_i}_{\chi k_i}$ are constans in $\mathscr{L}_{n+1}$
$P^i_j x_{k_1} \ldots x_{k_i}$ is a formula
If ϕ is a formula then $\neg \phi$ is a formula
If ϕ and φ are both formulas then $\phi \vee \varphi$ is a formula
If ϕ is a formula then $\forall^{\alpha}\phi$ is a formula, $\alpha < \gamma$
$\wedge, \rightarrow, \leftrightarrow$, and $\exists^{\alpha}$ can be defined as usual.

For limit $\alpha < \gamma$, $\text{Formu}_{\alpha} = \cup\{\text{Formu}_{\beta} : \beta < \alpha\}$
$\text{Formu} = \cup\{\text{Formu}_{\alpha} : \alpha < \gamma\}$

A sentence is a formula belonging to Formu without free variables.

[6] I will talk about the transformed axioms of Peano arithmetic in section 3.4.

3.2 Semantics

Now I will give the semantics of the language $\mathscr{L}_T$:

For any $\mathscr{L}_T$ model σ_T=<A_T, I_T, V_T>, A_T is a set, I_T is an explanation of predicates and constants and V_T is an assignment of the variables,

For any constant $c^{\alpha}_{\phi\Delta}$ [7], $I_T(c^{\alpha}_{\phi\Delta})$ is an element in A_T,

For any variable x_i, $V_T(x_i)$ is an element in A_T,

For any predicate P^i_j which is not T, I_T (P^i_j) is a subset of $(A_T)^i$ [8],

For predicate T, $I_T(T)=\{I_T(c^{\alpha}_{\phi 1})$: $\sigma(\phi)=1\}$,

For any formula $P^i_j c^{h_1}_{\phi k_1}\ldots\ c^{h_i}_{\phi k_i}$, $\sigma_T \models P^i_j c^{h_1}_{\phi k_1}\ldots\ c^{h_i}_{\phi k_i}$ iff $<\sigma_T(c^{h_1}_{\phi k_1})\ldots\sigma_T(c^{h_i}_{\phi k_i})>\in \sigma_T(P^i_j)$

For any formula $\neg\phi$, $\sigma_T \models \neg\phi$ iff $\sigma_T \not\models \phi$

For any formula $\phi \vee \varphi$, $\sigma_T \models \sigma \vee \varphi$ iff $\sigma_T \models \phi$ or $\sigma_T \models \varphi$

For any formula $\forall^{\alpha}\phi$, $\sigma_T \models \forall^{\alpha}x\phi$ iff for any $d\in M^{\alpha}$, $\sigma_T \models \phi(x/d)$, where $M^{\alpha}=\max\{M\subseteq A_T$: for any $\beta>\alpha$, $\sigma_T(c^{\beta}_{\phi\Delta})\notin M\}$

3.3 Axiomatization

The axioms:

(1) $\phi \to (\varphi \to \phi)$

(2) $(\phi \to (\varphi \to \chi)) \to ((\phi \to \varphi) \to (\phi \to \chi))$

(3) $(\neg\phi \to \varphi) \to ((\neg\phi \to \neg\varphi) \to \phi)$

(4) $\phi \to T(c^{\alpha+1}_{\phi 1})$ α is the largest ordinal appeared in ϕ

(5) $\forall^{\alpha}x(\phi \to \varphi) \to (\forall^{\alpha}x\phi \to \forall^{\alpha}x\varphi)$

(6) $\forall^{\alpha}x\phi \to \forall^{\beta}x\phi$ $\beta \le \alpha$

(7) $\phi \leftrightarrow \forall^{\alpha}x\phi$ x doesn't appear in ϕ

(8) $t\equiv t$

(9) $t\equiv t' \to (Pt_1\ldots t_{k-1}tt_{k+1}\ldots t_n \to Pt_1\ldots t_{k-1}t't_{k+1}\ldots t_n)$

(10) $\vdash \phi(x/t) \leftrightarrow \exists^{\alpha}x\phi(x)$ $t\neq c^{\beta}_{\phi\Delta}$ for all $\beta > \alpha$

(11) $\forall^{\alpha}x_1\ldots\forall^{\beta}x_n\phi$ ϕ is a formula with one of the form of (1)–(10)

The rule:

$\phi, \phi \to \varphi \vdash \varphi$ (Modus ponens)

The rules can be deduced: If $\Gamma \vdash A$ and $\Gamma \subseteq \Delta$ then $\Delta \vdash A$ (Structural rules)

If $\Gamma \vdash A$, $A \vdash B$ then $\Gamma \vdash B$ (Structural rules)

If $\Gamma, A \vdash C$, $B \vdash C$ then $\Gamma, A \vee B \vdash C$ (Disjunction elimination)

Theorems can be proved:

(12) $\vdash \phi \vee \neg\phi$ (Excluded Middle)

(13) $\phi \wedge \neg\phi \vdash \varphi$ (Explosion)

The following sentences aren't theorems:

$\forall^{\alpha}x\phi \to \phi(x/t)$ (t is a term)

$\phi(x/t) \to \forall^{\alpha}x\phi$ (t is a term)

$\exists^{\alpha}x\phi \to \phi(x/t)$ (t is a term)

$\phi(x/t) \to \exists^{\alpha}x\phi$ (t is a term)

[7] Δ=0 or Δ=1.

[8] $(A_T)^i$ is the Cartesian product of i A_Ts.

3.4 PA*

Because the language $\mathscr{L}_T$ doesn't have function symbols and the constants are special in my syntax I use the constant $c^0_{\perp 0}$ as the numeral 0, $c^0_{\perp i}$ as the numeral i, the predicate P^2_1 as the consequent relation, P^3_1 as the addition relation, P^3_2 as the multiplication relation. Then the axioms of PA are shown as follows:

(1) $\neg P^2_1 x_i c^0_{\perp 0}$

(2) $P^2_1 x_i x_j \wedge P^2_1 x_k x_j \rightarrow x_i = x_k$

(3) $P^3_1 x_1 c^0_{\perp 0} x_1$

(4) $P^2_1 x_2 x_3 \wedge P^3_1 x_1 x_2 x_4 \wedge P^2_1 x_4 x_5 \rightarrow P^3_1 x_1 x_3 x_5$

(5) $P^3_2 x_1 c^0_{\perp 0} c^0_{\perp 0}$

(6) $P^2_1 x_2 x_3 \wedge P^3_2 x_1 x_2 x_4 \wedge P^3_1 x_4 x_1 x_5 \rightarrow P^3_2 x_1 x_3 x_5$

Since the quantifiers change the induction axiom has to be transformed too:

$\forall^\alpha Q(Q(c^0_{\perp 0}|_\alpha) \wedge \forall^\alpha x_i \forall^\alpha x_j (Qx_i \wedge P^2_1|_\alpha (x_i x_j) \rightarrow Qx_j) \rightarrow \forall^\alpha x_i Qx_i)$

The formula $\forall^\alpha Q\phi$, as a second order quantified formula, means that for any subset $\sigma_T(Q)$ of M^α, ϕ holds, where $M^\alpha = \max\{M \subseteq A_T$: for any $\beta > \alpha$, $\sigma_T(c^\beta_{\phi\Delta}) \notin M\}$. $c^0_{\perp 0}|_\alpha$ is the least number in M^α, and $P^2_1|_\alpha$ $(x_i x_j)$ means x_j is the least number in M^α except x_i.

These transformed arithmetic axioms are all valid in the standard model.

Because of the change of the form of the induction axiom the Peano arithmetic is called PA* rather than PA in my system.

3.5 Soundness

Proof. It is straightforward to verify that the axioms of $\mathscr{L}_T$ are all true, and the rule preserves truth. ■

3.6 Completeness

The proof of the completeness of this system is almost the same as that of the first order logic[9]. The only differences are the definition of the set of witness and the steps relating to quantified sentences.

3.6.1 Definition

Let Γ be a set of consistent sentences. We claim that Γ contains witness of $\mathscr{L}_T$ if there is a set D of constants such that for any formula $\varphi(x)$ of $\mathscr{L}_T \cup D$ which has at most one free variable, if $\exists^\alpha x\varphi(x) \in \Gamma$ then there is a constant $d \in D$ such that $\{\varphi(x/d)\} \cup \{d \neq c^\beta_{\phi\Delta}: \beta > \alpha\} \subseteq \Gamma$. D is called the set of witness of Γ.

3.6.2 Lemma

Let W be a consistent theory of $\mathscr{L}_T$, D be a set of new constants such that $D \cap \mathscr{L}_T = \emptyset$, $\mathscr{L}' = \mathscr{L}_T \cup D$ and $|D| = |\mathscr{L}_T|$. There is a consistent set W' of sentences of $\mathscr{L}$' such that $W \subseteq W'$ and D is the set of witness of W'.

Proof. Suppose $\lambda = |D| = |\mathscr{L}_T|$ and $D = \{d_\xi : \xi < \lambda\}$. Let $\{\varphi_\xi : \xi < \lambda\}$ be the set of formulas of $\mathscr{L}_T \cup D$ with at most one free variable. Then we define sets W_ξs, $\xi < \lambda$, as follows:

[9] Cf. C. C. Chang & H. J Keisler, 1990, *Model Theory*, New York: Elsevier Publishing Company, INC. §2.1.

(1) W_0=W.

(2) $W_{\xi+1}$= $W_\xi \cup \{\exists^\alpha x\varphi_\xi(x)\} \cup \{\varphi_\xi(x/d_\eta)\} \cup \{d_\eta \neq c^\beta_{\phi\Delta}: \beta > \alpha\}$ if $\{\exists^\alpha x\varphi_\xi(x)\} \cup W_\xi$ is consistent, where d_η is the first constant which doesn't appear in W_ξ and φ_ξ;

$W_{\xi+1}$= W_ξ if $\{\exists^\alpha x\varphi_\xi(x)\} \cup W_\xi$ is inconsistent.

(3) For limit ξ, $W_\xi = \cup_{\beta<\xi} W_\beta$.

Obviously, $W_0 \subseteq W_1 \subseteq$.... Let W'=$\cup_{\xi<\lambda} W_\xi$.

Next I will prove inductively that W' is consistent.

(1) W_0=W, according to the hypothesis W_0 is consistent.

(2) Suppose W_ξ is consistent. If $\{\exists^\alpha x\varphi_\xi(x)\} \cup W_\xi$ is inconsistent then $W_{\xi+1}$= W_ξ. Hence $W_{\xi+1}$ is consistent. If $\{\exists^\alpha x\varphi_\xi(x)\} \cup W_\xi$ is consistent then $W_\xi \cup \{\exists^\alpha x\varphi_\xi(x)\} \cup \{d_\eta \neq c^\beta_{\phi\Delta}: \beta > \alpha\}$ is consistent. Assume it's not the case i.e. $W_\xi \cup \{\exists^\alpha x\varphi_\xi(x)\} \cup \{d_\eta \neq c^\beta_{\phi\Delta}: \beta > \alpha\}$ is inconsistent. Because d_η is a new constant there must be $\lambda > \alpha$ such that $W_\xi \cup \{\exists^\alpha x\varphi_\xi(x)\} \vdash \forall^\lambda x(x=c^\lambda_{\phi\Delta})$. Since $W_\xi \cup \{\exists^\alpha x\varphi_\xi(x)\} \vdash \exists^\alpha x\varphi(x)$. Hence we have $W_\xi \cup \{\exists^\alpha x\varphi_\xi(x)\} \vdash \forall^\lambda x(x=c^\lambda_{\phi\Delta}) \wedge \exists^\alpha x\varphi(x)$. However, $\forall^\lambda x(x=c^\lambda_{\phi\Delta}) \wedge \exists^\alpha x\varphi(x)$ is an absurdity which is false under every $\mathscr{L}_T$ model. Hence $W_\xi \cup \{\exists^\alpha x\varphi_\xi(x)\}$ is inconsistent. This contradict with the hypothesis. So $W_\xi \cup \{\exists^\alpha x\varphi_\xi(x)\} \cup \{d_\eta \neq c^\beta_{\phi\Delta}: \beta > \alpha\}$ is consistent. Since $\{d_\eta \neq c^\beta_{\phi\Delta}: \beta > \alpha\} \vdash \exists^\alpha x\varphi_\xi(x) \rightarrow \varphi_\xi(x/d_\eta)$ is valid $W_\xi \cup \{\exists^\alpha x\varphi_\xi(x)\} \cup \{d_\eta \neq c^\beta_{\phi\Delta}: \beta > \alpha\} \vdash \exists^\alpha x\varphi_\xi(x) \rightarrow \varphi_\xi(x/d_\eta)$ is valid too. And then $W_\xi \cup \{\exists^\alpha x\varphi_\xi(x)\} \cup \{d_\eta \neq c^\beta_{\phi\Delta}: \beta > \alpha\} \vdash \varphi_\xi(x/d_\eta)$ is valid. Hence $W_\xi \cup \{\exists^\alpha x\varphi_\xi(x)\} \cup \{\varphi_\xi(x/d_\eta)\} \cup \{d_\eta \neq c^\beta_{\phi\Delta}: \beta > \alpha\}$ is consistent.

(3) For limit λ, if W_α is inconsistent then there is a $\alpha < \lambda$ such that W_α is inconsistent. However, by induction hypothesis, for any $\alpha < \lambda$, W_α is consistent. So W_λ is consistent. ■

3.6.3 Lemma

Suppose W is a consistent theory of $\mathscr{L}_T$, D a set of new constants. Then W has a model $\mathfrak{A}$ such that every element of $\mathfrak{A}$ is an explanation of a constant in D∪C i.e. A=$\{a^{\mathfrak{A}}: a \in D\cup C\}$ where C is the set of constants in $\mathscr{L}_T$, and $\mathfrak{A} \models$W.

Proof. According to Lindenbaum Theorem W can be extended to a maximal consistent set W' of sentences of $\mathscr{L}_T$. Then there is a language $\mathscr{L}'_T$ such that $\mathscr{L}'_T = \mathscr{L}_T \cup D$, $D \cap \mathscr{L}_T = \varnothing$ and $|D| = |\mathscr{L}_T|$. In line with 3.6.1 W' can be extended to a set W" of sentences of $\mathscr{L}_T$' with D as the set of witness. Then we can extended W" to a maximal consistent set W* of sentences of $\mathscr{L}_T$' with D as the set of witness according to Lindenbaum Theorem.

Defining the binary relation ~ as follows: for any $a_1, a_2 \in D\cup C$, $a_1 \sim a_2$ iff $T \vdash a_1 = a_2$. For $a \in D\cup C$, ã={a'∈D∪C: a∼a'} i.e. ã is the equivalence class of the constant a. Defining a model $\mathfrak{A} = \langle A, I\rangle$ as follows: A={ã: a∈D∪C}, for any a∈D∪C, I(a)=ã; for any predicate P^n_i and $\widetilde{a_1}...\widetilde{a_n} \in A$

$$W^* \vdash P^n_i(a_1...a_n) \text{ iff } \mathfrak{A} \models \varphi[\widetilde{a_1}...\widetilde{a_n}]$$

Next we will prove inductively that for any formula $\varphi(x_1...x_n)$ of $\mathscr{L}_T \cup D$ and $\widetilde{a_1}...\widetilde{a_n} \in A$,

$$W^* \vdash \varphi(a_1...a_n) \text{ iff } \mathfrak{A} \models \varphi[\widetilde{a_1}...\widetilde{a_n}]$$

(1) If $\varphi(x_1...x_n)$ is atomic formula $P^n_i(x_1...x_n)$, according to the explanation of the model for predicate we have $a_1...a_n \in D\cup C$ such that

$$W^* \vdash P_i^n(a_1...a_n)^{10} \text{ iff } \mathfrak{A} \models P_i^n(x_1...x_n)[\widetilde{a_1}...\widetilde{a_n}] \text{ iff } \mathfrak{A} \models \varphi[\widetilde{a_1}...\widetilde{a_n}]$$

(2) It is easy to prove that the equivalence holds when $\varphi(x_1...x_n)$ is a negative formula or a disjunctive formula.

(3) Suppose $\varphi(x_1...x_n)$ is a formula with the form $\exists^\alpha y\psi(x_1...x_n\ y)$. If $\mathfrak{A} \models \varphi[\widetilde{a_1}...\widetilde{a_n}]$, then there is a $\tilde{d} \in M^\alpha$ such that $\mathfrak{A} \models \psi[\widetilde{a_1}...\widetilde{a_n}\ \tilde{d}]$. According to the induction hypothesis we have $W^* \vdash \psi(a_1...a_n\ d)$. Since $\tilde{d} \in M^\alpha = \max\{M \subseteq A_T$: for any $\beta>\alpha$, $\sigma_T(c^\beta_{\phi\Delta}) \notin M\}$ we have $\tilde{d} \neq \widetilde{c^\beta_{\phi\Delta}}$ for any $\beta > \alpha$. And then $d \neq c^\beta_{\phi\Delta}$ for any $\beta > \alpha$. Hence $W^* \vdash \exists^\alpha y\psi(a_1...a_n\ y)$[11]. If $W^* \vdash \exists^\alpha y\psi(a_1...a_n\ y)$, then in accordance with the definition 3.6.1 there is a $d \in D \cup C$ such that $\{\exists^\alpha y\psi(a_1...a_n\ y)\} \cup \{\psi(a_1...a_n\ d)\} \cup \{d \neq c^\beta_{\phi\Delta}: \beta > \alpha\} \subseteq W^*$. Then we get $W^* \vdash \psi(a_1...a_n\ d)$ and $W^* \vdash d \neq c^\beta_{\phi\Delta}$ for any $\beta > \alpha$. By induction hypothesis $\mathfrak{A} \models \psi(a_1...a_n\ d)$ and $\mathfrak{A} \models d \neq c^\beta_{\phi\Delta}$ for any $\beta > \alpha$. Hence $\tilde{d} \in M^\alpha$ and then $\mathfrak{A} \models \exists^\alpha y\psi(a_1...a_n y)$. ■

3.6.4 Completeness theorem

Suppose Σ is a set of formulas of $\mathscr{L}_T$ and φ a formula of $\mathscr{L}_T$. If $\Sigma \models \varphi$ then $\Sigma \vdash \varphi$. Especially, if $\models \varphi$ then $\vdash \varphi$. And if Σ is consistent then there is a model of Σ.

Proof. Suppose $\Sigma \models \varphi$ but $\Sigma \nvdash \varphi$, then obviously $\Sigma \cup \{\neg \varphi\}$ is consistent. Let E be a set of new constants such that $|E| = |\mathscr{L}_T|$ and $E \cap \mathscr{L}_T = \emptyset$. Then arrange the formulas in $\Sigma \cup \{\neg \varphi\}$ in a linear order. And then substitute the new constants in E for the variables in $\Sigma \cup \{\neg \varphi\}$ in such a way that in the same formula we use the same constant to substitute for the same variable, different constants for different variables, in different formulas we use constants in E that haven't appeared in previous formulas to substitute, for sentences in Σ we reserve their form. Let $W \cup \{\neg \varphi'\}$ be the set of sentences replacing the formulas of $\Sigma \cup \{\neg \varphi\}$ in the above way, therefore $W \cup \{\neg \varphi'\}$ is a set of sentences of $E \cup \mathscr{L}_T$. Let D be an arbitrary set of new constants such that $D \cap \mathscr{L}_T = E \cap \mathscr{L}_T = D \cap E = \emptyset$ and $|D| = |\mathscr{L}_T|$. In accordance with lemma 3.6.2 $W \cup \{\neg \varphi'\}$ can be extended to a consistent set W' with D as the set of witness. Then according to lemma 3.6.3 W' has a model $\mathfrak{A}'$. Let $\mathfrak{A}$ be $\mathfrak{A}'|_{\mathscr{L}_T \cup E}$, then $\mathfrak{A} \models W \cup \{\neg \varphi'\}$. And then there is a model $\mathfrak{B}$ such that for variable x_i appears in a formula in $\Sigma \cup \{\neg \varphi\}$, let $(x_i)^{\mathfrak{B}} = (e_i)^{\mathfrak{A}}$ where e_i is the constant replacing x_i in the corresponding formula in $W \cup \{\neg \varphi'\}$. For constant t and predicate Q, let $t^{\mathfrak{B}} = t^{\mathfrak{A}}$ and $Q^{\mathfrak{B}} = Q^{\mathfrak{A}}$. Obviously, for any formula ψ in $\Sigma \cup \{\neg \phi\}$, $\mathfrak{B} \models \psi$. Therefore there is a model $\mathfrak{B}$ such that $\mathfrak{B} \models \Sigma$ and $\mathfrak{B} \models \neg \varphi$ i.e. $\mathfrak{B} \models \Sigma$ and $\mathfrak{B} \not\models \varphi$. However, this contradicts with $\Sigma \models \varphi$. So if $\Sigma \models \varphi$ then $\Sigma \vdash \varphi$. ■

The usual understanding of (T) schema is like this: $T(\overline{\ulcorner\phi\urcorner}) \leftrightarrow \phi$. Here $\overline{\ulcorner\phi\urcorner}$ refers to the Gödel number of the sentence ϕ. However, in my proposal, I will change the schema a little. My (T) schema is like this: $T(c^{\alpha+1}_{\phi 1}) \leftrightarrow \phi$ (α is the largest superscript appears in ϕ). In order to distinguish the two schemas I will call my schema (T^*) schema. It seems that there is no difference between the two schemas except the change of the symbol. However, you will see that the meaning of the change is not the variation of the symbols but that the (T^*) schema, not the (T_n) schemas, will be valid under certain condition without the trouble of the liar paradox.

Obviously, according to the explanation of the predicate T in the semantics, the (T^*) schema isn't valid. For example, let τ be a $\mathscr{L}_T$-model. Then let $\tau(c^\alpha_{\phi 1}) = \tau(c^\alpha_{\neg\phi 1})$.

[10] Because there are no functions in my system we avoid the trouble of them.
[11] cf. (10) in 3.3 axiomatization

If (T^*) schema hold, we will get $\phi \leftrightarrow \neg \phi$ from the (T^*) schema and the explanation of T, which is a contradiction. So we need some restrictions on the (T^*) schema.

3.7 Definition

Let σ_{T^*} be a model class, σ_{T^*} ={σ_T: for any constants $c^{\alpha}_{\phi\Delta}$, $c^{\beta}_{\varphi\Delta}$, if σ_T ($c^{\alpha}_{\phi\Delta}$)=σ_T($c^{\beta}_{\varphi\Delta}$) then $\phi \leftrightarrow \varphi$}.

3.8 Theorem

The (T^*) schema is valid in the model class σ_{T^*}.
Proof. We already have $\phi \to$ T($c^{\alpha+1}_{\phi 1}$), α is the largest superscript appears in ϕ, as our axiom, so we need only prove the right to left side of the equivalence. If this implication is false, then we have a model τ_T belonging to σ_{T^*} such that $\tau_T \models$ T($c^{\alpha+1}_{\phi 1}$) but $\tau_T \not\models \phi$. Since $\tau_T \models$ T($c^{\alpha+1}_{\phi 1}$), according to the semantics of the language, τ_T($c^{\alpha+1}_{\phi 1}$)$\in$ τ_T(T). Hence there is a constant $c^{\beta}_{\varphi 1}$ such that τ_T($c^{\beta}_{\varphi 1}$)$\in$ τ_T(T), $\tau_T(\varphi)$=1, and τ_T($c^{\beta}_{\varphi 1}$) =τ_T($c^{\alpha+1}_{\phi 1}$). Then, according to the special requirement of the model class σ^*_T, we have $\tau_T(\phi)$=$\tau_T(\varphi)$=1. Hence $\tau_T \models \phi$. This is contradictory to the hypothesis $\tau_T \not\models \phi$. So $\sigma^*_T \models \phi \leftrightarrow$ T($c^{\alpha+1}_{\phi 1}$) (α is the largest superscript appears in ϕ). ■

3.9 Theorem

Let γ=ω. For any formula ϕ of $\mathscr{L}_T$ and any $\tau \in \sigma_{T^*}$ there is a translation function Tran() from $\mathscr{L}_T$ to $\mathscr{L}_{Tarski}$ such that

$$\text{if } \tau \models \phi, \text{ then } \tau \models \text{Tran}(\phi)$$

Proof. First let us define a translation from $\mathscr{L}_T$ to $\mathscr{L}_{Tarski}$.

(1)Tran($\perp$)=$t_1 \neq t_1$

(2)For any predicate $P^i_n \neq$ T, Tran($P^i_n x_i$)= $P^i_n x_i$

(3)For any predicate $P^i_n \neq$ T, Tran(P^i_n($c^{\alpha}_{\phi\Delta}$))=P^i_n(Tran($\overline{\ulcorner \phi \urcorner}$))

(4)For predicate T, if x_i is not bounded by quantifier, Tran(T(x_i))= $T^{\alpha}(x_i)$, if there is a β such that τ($c^{\beta}_{\phi\Delta}$)=$\tau(x_i)$ and α=min{β: τ($c^{\beta}_{\phi\Delta}$)=$\tau(x_i)$}; if there is not a β such that τ($c^{\beta}_{\phi\Delta}$)=$\tau(x_i)$, Tran(T(x_i))=$t_2 \neq t_2$

(5)For predicate T, if x_i is bounded by $\underline{\text{quantifier}}$ $\forall^{\beta} x_i$ then Tran(T(x_i))=$T^{\beta}(x_i)$

(6)For predicate T, Tran(T($c^{\alpha}_{\phi\Delta}$))=T^{α}($\overline{\ulcorner \text{Tran}(\phi) \urcorner}$)

(7) Tran($\neg \phi$)=$\neg$Tran(ϕ)

(8) Tran($\phi \vee \varphi$)=Tran(ϕ)$\vee$Tran(φ)

(9) Tran($\forall^n x\phi$)=$\forall x_i \in Q_n$Tran(ϕ)

Obviously, if ϕ is a formula of $\mathscr{L}_n$ in the construction of the language $\mathscr{L}_T$, then there is a m$\leqslant$n such that Tran(ϕ) is a formula of $\mathscr{L}_m$ in the construction of $\mathscr{L}_{Tarski}$.

Then we can prove the theorem by induction.

Let $\tau(Q_{\alpha})$=max{Q$\subseteq A_T$: for any β>α,τ($c^{\beta}_{\phi\Delta}$) $\notin$Q}. Let τ($c^{\alpha}_{\phi\Delta}$)=τ(Tran($\overline{\ulcorner \phi \urcorner}$))

(1) Consider the atomic formula,

For predicate $P^i_n \neq$T, if $\tau \models P^i_n(x_i)$, because Tran($P^i_n x_i$)= $P^i_n x_i$, $\tau \models$Tran ($P^i_n x_i$)

For predicate $P^i_n \neq$ T, if $\tau \models P^i_n$($c^{\alpha}_{\phi\Delta}$), then τ($c^{\alpha}_{\phi\Delta}$)$\in$ $\tau(P^i_n)$, since τ($c^{\alpha}_{\phi\Delta}$)=τ($\overline{\ulcorner \text{Tran}(\phi) \urcorner}$), $\tau \models P^i_n$($\overline{\ulcorner \text{Tran}(\phi) \urcorner}$).

For predicate T and variable x_i which isn't bounded by quantifier, if $\tau \models T(x_i)$ then according to the definition of T there is a least β such that $\tau(x_i)=\tau(c^{\beta}_{\phi 1})$, then according to the definition of the translation $Tran(T(x_i))= T^{\beta}(x_i)$. According to the construction of the language $\mathscr{L}_T$, there is a ϕ in $\mathscr{L}_{\beta-1}$ such that $\tau \models \phi$. By induction hypothesis $\tau \models Tran(\phi)$. Obviously, $Tran(\phi)$ is a formula of $\mathscr{L}_{\beta-1}$ of Tarski's hierarchies. Then according to the definition of T^{β}, $\tau \models T^{\beta}(\overline{\ulcorner Tran(\phi)\urcorner})$. Since $\tau(x_i)=\tau(c^{\beta}_{\phi 1})$ and $\tau(c^{\alpha}_{\phi\Delta})=\tau(Tran(\overline{\ulcorner\phi\urcorner}))$ $\tau \models T^{\beta}(x_i)$.

For predicate T and constant $c^{\alpha}_{\phi\Delta}$, if $\tau \models T(c^{\alpha}_{\phi\Delta})$ then according to the requirement of the model class σ^*_T, $\tau \models \phi$. Through induction hypothesis, $\tau \models Tran(\phi)$. In line with the construction of the two languages ϕ and $Tran(\phi)$ belong to $\mathscr{L}_{\alpha-1}$ of each language. Hence $\tau(\overline{\ulcorner Tran(\phi)\urcorner}) \in \tau(T^{\alpha})$. Therefore $\tau \models T^{\alpha}(\overline{\ulcorner Tran(\phi)\urcorner})$

(2) The Boolean cases when $\phi=\neg\varphi$ and $\phi=\varphi\vee\chi$ are easy to prove by induction hypothesis.

(3) $\phi=\forall^{\alpha}x\varphi$. Suppose $\tau \models \forall^{\alpha}x\varphi$. According to the semantics of the $\mathscr{L}_T$, for any $d\in M^{\alpha}$, $\sigma_T \models \phi(x/d)$, where $M^{\alpha}=\max\{M\subseteq A_T$: for any $\beta>\alpha$, $\sigma_T(c^{\beta}_{\phi\Delta k_i})\notin M\}$, by induction hypothesis $\tau \models Tran(\phi)$, so $\tau \models \forall x_i \in Q_n Tran(\phi)$. ■

This theorem is about the relationship between my proposal and Tarski's hierarchies. It asserts that in some sense i.e. when $\gamma=\omega$ and (T^*) schema holds, $\mathscr{L}_T$ is a fragment of $\mathscr{L}_{Tarski}$ i.e. its expressive power is no more than that of $\mathscr{L}_{Tarski}$'s. In fact, in this sense, the expressive power of $\mathscr{L}_T$ is less than that of $\mathscr{L}_{Tarski}$'s because there are some sentences of $\mathscr{L}_{Tarski}$ that cannot be translated into sentences of $\mathscr{L}_T$, such as $\exists x\phi(x)$ cannot be translated into $\mathscr{L}_T$ sentence.

4 Examination

In this section I will use the examples mentioned in section 1 to check my proposal.

First, I want to say something about the example (5). I don't think that the contingent paradox is a real problem. I think that depending on environments is just like depending on models, the same sentence is or isn't a paradox because of different environments is just like the same sentence has different values under different models. In the strict artificial language the "environment" is fixed, so whether a sentence is a paradox is also fixed. It's not our work to make sure whether a sentence is a paradox or not. What we have to do is to treat it when we know a sentence is a paradox. Suppose (1) and (2) in this example are both paradoxes. Then this example is like the example (2). We leave the solution to this example until we handle example (2).

Let's look at the example (1)

(1): (1) is not true

or using my language $\mathscr{L}_T$:

$$c^{\alpha}_{\phi\Delta}: \neg T\,(c^{\alpha}_{\phi\Delta})$$

Remember the reasoning in section 1. Suppose $c^{\alpha}_{\phi\Delta}$ is not true. Then we have $\neg T(c^{\alpha}_{\phi\Delta})$. If we want to get the contradiction we must use the intersubstitutivity and the (T) schema. However, the (T) schema here is (T^*) schema i.e. $T(c^{\alpha}_{\phi 1})\leftrightarrow\phi$. And this schema is not valid universally but valid in a special model class. What is valid universally here is $\phi \rightarrow$ T($c^{\alpha}_{\phi 1}$). Through it, we can get $\neg\ \phi$. In order to get the contradiction we have to suppose ϕ is $\neg$T ($c^{\alpha}_{\phi\Delta}$). This is impossible according to the construction of the language $\mathscr{L}_T$. The process of the construction of $\mathscr{L}_T$ shows that we first have the sentence ϕ in $\mathscr{L}_\alpha$ then we have the constant $c^{\alpha+1}_{\phi 1}$ in $\mathscr{L}_{\alpha+1}$. So ϕ cannot be a sentence containing itself as a part in it. Suppose $c^{\alpha}_{\phi\Delta}$ is true. In order to get the contradiction we need $\vdash$T(a)$\rightarrow\ \phi$ where a is the name of ϕ. However this is not valid here. (Even if we talk about this example under the model class σ_{T*}, therefore we can get $\vdash$T($c^{\alpha}_{\phi\Delta}$)$\rightarrow\ \phi$, we cannot conclude the contradiction because of the construction of the language $\mathscr{L}_T$).

Example (2):

$$c^{\alpha}_{\phi 1}\text{: } c^{\beta}_{\varphi 1} \text{ is not true}$$
$$c^{\beta}_{\varphi 1}\text{: } c^{\alpha}_{\phi 1} \text{ is true}$$

Let's start from $c^{\beta}_{\phi 1}$ *is not true* i.e. $\neg$T($c^{\beta}_{\varphi 1}$). Then if we want to get $\neg$($c^{\alpha}_{\phi 1}$ is true) we have to use (T) schema. However what is valid is $\neg$T($c^{\alpha}_{\phi 1}$)$\rightarrow\neg\ \phi$ i.e. $\phi \rightarrow$ T($c^{\alpha}_{\phi 1}$). So we can only get $\neg\ \varphi$ from $\neg$T($c^{\beta}_{\varphi 1}$). In order to construct the paradox, φ must be the sentence $c^{\alpha}_{\phi 1}$ *is true*. Based on the construction of the language $\mathscr{L}_T$ we know $\beta > \alpha$. And then in order to get a contradiction we must go through from $\neg$($c^{\alpha}_{\phi 1}$ is true) to $\neg$ ($c^{\beta}_{\varphi 1}$ is not true) i.e. we have to use T(a)$\rightarrow\ \phi$ where a is the name of ϕ. But this is not valid in our system. (Even under the model class σ_{T*}, we cannot get a paradox because of the construction of the language). The same is true of example (5) when we suppose it is a paradox.

Example (3), Curry paradox. Let's first translate the Curry sentence into our language. Let ϕ be equivalent to the next sentence

$$\text{T}(c^{\alpha}_{\phi 1}) \rightarrow \perp$$

This is not possible according to the construction of the language $\mathscr{L}_T$. Because $c^{\alpha}_{\phi 1}$ has already appeared in the sentence, ϕ have to be a sentence appeared in $\mathscr{L}_{\alpha-1}$. If we let ϕ be the sentence T($c^{\alpha}_{\phi 1}$) $\rightarrow\perp$, then we have $c^{\alpha+1}_{\phi}$ in $\mathscr{L}_{\alpha+1}$. However, this is not possible according to the construction of the language. Hence we have to let φ be T($c^{\alpha}_{\phi 1}$) $\rightarrow\perp$. If we want to infer as we do in the example we have to use (T) schema in the 2nd step of the deduction. But this is not valid since the implication T(a)$\rightarrow\ \phi$ is not valid in this proposal. Therefore the inference of this example is not true.

Example (4), the Yablo's paradox. First the Yablo's paradox cannot be generated in my language because the rule $\forall^-$ is not valid in my proposal. This is a weakness of $\mathscr{L}_T$'s expressive power. However, even if we suppose that we have that rule, this paradox can't emerge. The key step in the inference is the one from S_k *is untrue* to $\neg S_k$. This step is legal in my proposal. According to the construction of the language $\mathscr{L}_T$, for any (S_α) and (S_β), if $\alpha > \beta$ then (S_α) appears in a lower language $\mathscr{L}_\alpha$ while (S_β) appears in a higher language $\mathscr{L}_\beta$. If, because of the construction of the language $\mathscr{L}_T$, the sequence is infinite then we will have an infinite descend chain which contradicts with the well-ordering theorem. And hence it contradicts with the construction of the language $\mathscr{L}_T$.

Example (6), the Tarski's undefinability theorem. The key of the proof of this theorem is the construction of the Gödel sentence. First we need an open formula with one variable like this

$$\exists x(\text{ x is the self-application of v} \wedge P(x))$$

where P(x) is any open formula with x as the only free variable, and the "self-application" means substituting the name (or the Gödel number) of the open formula for all free occurrence of v in it. Then let D(v) be the open formula and $\overline{\ulcorner D(v) \urcorner}$ be the name of it. Let S be the self-application of D(v), i.e. the sentence

$$\exists x(\text{x is the self-application of } \overline{\ulcorner D(x) \urcorner} \wedge P(x))$$

But $\overline{\ulcorner S \urcorner}$, the name of S, is the unique self-application of $\overline{\ulcorner D(x) \urcorner}$. Then we can get

$$S \leftrightarrow \exists x(x=\overline{\ulcorner S \urcorner} \wedge P(x))$$

And hence

$$S \leftrightarrow P(\overline{\ulcorner S \urcorner})$$

If we change the predicate P() into $\neg$T() then together with Tarski's (T) schema, we can get $T(\overline{\ulcorner S \urcorner}) \leftrightarrow \neg T(\overline{\ulcorner S \urcorner})$. This is a contradiction.

However, this construction doesn't hold in our proposal. Because we have many existential quantifiers $\exists^{\alpha}$s rather than a $\exists$, the well-formed formula is

$$\exists^{\alpha} x(\text{x is the self-application of v} \wedge P(x))$$

so after substituting $\overline{\ulcorner D(v) \urcorner}$ (or $c_{\phi 0}^{\alpha+1}$) for v we get

$$\exists^{\alpha} x(\text{x is the self-application of } \overline{\ulcorner D(v) \urcorner} \wedge P(x))$$

In order to use the theorem related to predicate T, we need the constant $c_{\phi 0}^{\alpha+1}$ instead of $\overline{\ulcorner D(v) \urcorner}$ where ϕ is the formula $\exists^{\alpha}$x(x is the self-application of v $\bigwedge$P(x)). Then we have

$$\phi \leftrightarrow \exists^{\alpha} x(x=c_{\phi 0}^{\alpha+1} \wedge P(x))$$

The sentence $\exists^{\alpha} x(x=c_{\phi 0}^{\alpha+1} \wedge P(x))$ is always false since there is no d that is $\sigma_T(c_{\phi 0}^{\alpha+1})$ in the max{$M \subseteq A_T$: for any $\beta > \alpha$, $\sigma_T(c_{\phi\Delta k_i}^{\beta}) \notin M$} according to our semantic explanation of existential sentence. Therefore we cannot use $P(c_{\phi 1}^{\alpha+1})$ as the abbreviation of $\exists^{\alpha} x(x=c_{\phi 1}^{\alpha+1} \wedge P(x))$. Hence, after substituting $\neg$T() for P(), we can't get the contradiction $\phi \leftrightarrow \neg T(c_{\phi 1}^{\alpha+1})$. But we can get

$$\exists^{\alpha} x(x=c_{\phi 1}^{\alpha+1} \wedge \neg T(x)) \leftrightarrow \exists^{\alpha} x(x=c_{\phi 1}^{\alpha+1} \wedge T(x))$$

because they are both false.

5 Conclusion

In conclusion. The (T) schema is not valid universally, but valid only in a model class. I haven't denied direct or indirect self-reference. What I do is that the self-reference sentence cannot be used in the theorem related to T predicate i.e. $\vdash \phi \rightarrow T(c_{\phi 1}^{\alpha})$ or in reverse this theorem cannot be used in self-reference sentence. We can construct self-reference sentence, such as "this sentence is written by English", and use the theorem separately. But, we can't use them at the same time. Because the other side of the (T) schema i.e. $T(c^{\alpha}_{\phi 1}) \rightarrow \phi$ doesn't hold universally in my proposal the problems of the liar paradox and the liar-like paradox i.e. the Yablo's paradox which lies in the conjunction of self-reference and (T) schema is solved. And the Tarski's undefinability theorem which also lies in the construction of the sentences and (T) schema is avoided. Through changing the construction of the language and the form of the (T) schema we successfully avoid the paradoxes and the undefinability theorem while we have only one truth predicate rather than many true_ns and our semantics is bivalent.

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